A Markov property for sample paths of stochastic processes

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We prove a graphical Markov property for sample paths of various discrete- and continuous-time stochastic processes. When a dynamical system is modelled by a set of (ordinary, partial, or stochastic) differential equations, the existence and uniqueness of solutions can yield the existence of a (deterministic) solution function, that takes initial conditions and sample paths of exogenous noise processes as input, and maps them to the sample path of the solution of the differential equation. Similarly, sample paths of variables of a dynamic Bayesian Network can be expressed as measurable (deterministic) functions of sample paths, of other variables, and an exogenous noise process. This approach allows to model the sample paths of various stochastic processes with SCMs, where the variables are entire sample paths of the process, and the structural equations are the measurable solution functions as described above. For the graph of this SCM, existing work by Forré, Mooij and Bongers yields a Markov property. This provides a 'global' alternative to the local independence graph as developed by Didelez, Mogensen and Hansen. Our Markov property implies a do-calculus for interventions on the level of entire sample paths. Combined with recent developments in conditional independence testing for functional data, this might be a promising approach for 'global' causal discovery and causal reasoning for time series data.